Int. J. Heat Mass Transfer. Vol. 20, pp. 1173–1184. Pergamon Press 1977. Printed in Great Britain

NATURAL CONVECTION HEAT TRANSFER FROM HORIZONTAL CYLINDERS TO AIR, WATER AND SILICONE OILS FOR RAYLEIGH NUMBERS BETWEEN 3×10^2 AND 2×10^7

R. M. FAND, E. W. MORRIS and M. LUM
Department of Mechanical Engineering, University of Hawaii, Honolulu, Hawaii 96822, U.S.A.

(*Received* 18 *October* 1976 *and in revised form* 26 *January* 1977)

Abstract—This paper presents the results of an experimental investigation of heat transfer by natural convection from horizontal cylinders to air, water and three silicone oils. The experimental range of the Rayleigh number was from 2.5×10^2 to 1.8×10^7 and the Prandtl number varied from 0.7 to 3090. On the basis of the experimental data gathered here, three correlation equations have been determined. These equations have the same algebraic form and differ only in the method used in each to evaluate fluid properties as a function of temperature. The influence of viscous dissipation is discussed.

NOMENCLATURE

C,	numerical constant;
c_p,	specific heat at constant pressure;
d,	diameter of cylindrical test specimen;
E,	percent error: $E = 100(Nu_{\text{cal}} - Nu_{\text{exp}})/Nu_{\text{exp}}$;
E_m,	percent mean error: $E = \sum_{i=1}^{n} E_i/n$;
E_{md},	percent mean deviation of error: $E_{md} = \sum_{i=1}^{n} \lvert E_i \rvert/n$;
E_{sd},	percent standard deviation of error: $E_{sd} = \left\{\left[\left(\sum_{i=1}^{n} E_i^2\right) - \left(\sum_{i=1}^{n} E_i\right)^2 \Big/ n\right]\Big/ (n-1)\right\}^{1/2}$;
Gr,	Grashof number: $Gr = g\beta\Delta t\, \mathrm{d}^3 \nu^{-2}$;
g,	gravitational acceleration;
h,	heat-transfer coefficient: $h = q(\Delta t)^{-1}$;
j,	a number between zero and one;
k,	thermal conductivity;
m,	numerical constant;
n,	number of experimental data points;
Nu,	Nusselt number: $Nu = h\,\mathrm{d}k^{-1}$;
Pr,	Prandtl number: $Pr = c_p\mu k^{-1}$;
Q,	electric power delivered to the test section of the test cylinder;
q,	heat-transfer per unit area per unit time;
t,	temperature;
Δt,	difference between the surface temperature of the test specimen and the bulk temperature of the fluid: $\Delta t = (t_s - t_b)$;
Ra,	Rayleigh number: $Ra = GrPr$.

Greek symbols

β,	coefficient of thermal expansion;
Δ,	denotes a change in a quantity; thus $\Delta(\Delta t)$ denotes a change in Δt;
μ,	dynamic viscosity;
ν,	kinematic viscosity;
ρ,	density.

Subscripts

b,	refers to conditions in the fluid outside of the thermal boundary layer;
cal,	refers to quantities calculated from the correlation equations;
exp,	refers to quantities determined from experimental data;
f,	refers to fluid properties evaluated at the mean film temperature: $t_f = t_b + 0.5(t_s - t_b)$;
g,	surface correction exponent on the Grashof number;
j,	refers to fluid quantities evaluated at some reference temperature: $t_j = t_b + j(t_s - t_b)$; $0 \leqslant j \leqslant 1$;
n,	surface correction exponent on the Nusselt number;
p,	surface correction exponent on the Prandtl number;
s,	refers to conditions at the heat-transfer surface;
y,	surface correction exponent on the Rayleigh number.

INTRODUCTION AND STATEMENT OF OBJECTIVE

IN 1881 LORENZ [1] presented an analysis of heat transfer by natural convection from a vertical plate immersed in an infinite medium. He assumed that the flow within the convective layers was primarily parallel to the surface of the plate and that the thermophysical properties of the fluid were constant. His resulting correlation equation was

$$Nu = 0.548Ra^{0.25}. \tag{1}$$

In 1942 McAdams [2] analyzed experimental data of many investigators to correlate the average natural convection heat-transfer coefficient for horizontal cylinders. He arrived at an equation similar to Lorenz's, namely,

$$Nu_f = CRa_f^m \tag{2}$$

where the values of C and m depend on the range of Ra_f. The subscript f in equation (2) signifies that the fluid properties are evaluated at the mean film temperature defined by

$$t_f = t_b + j(t_s - t_b); \quad j = 0.5. \tag{3}$$

Kutateladze [3] has modified Lorenz's formula on the basis of an empirical analysis of many experimental data. According to Kutateladze, the following correlation equation is applicable to horizontal cylinders in the range $5 \times 10^2 \leqslant Ra_f \leqslant 2 \times 10^7$:

$$Nu_f = K(Pr_f)Ra_f^{0.25} \tag{4}$$

where $K(Pr_f) = 0.54$ for $0.5 \leqslant Pr_f \leqslant 200$ (referred to hereinafter as the "low-*Pr* range") and $K(Pr_f) = 0.65$ for $Pr_f > 200$ (referred to hereinafter as the "high-*Pr* range").

Morgan [4], after making an extensive survey of the literature on natural convection from horizontal cylinders up to and including 1974, has recommended the following correlation:

$$Nu_f = CRa_f^m \tag{5}$$

where the values of C and m depend upon the range of Ra_f as follows:

Range of Ra_f	C	m
10^{-4} *to* 10^{-2}	0.675	0.058
10^{-2} *to* 10^{2}	1.02	0.148
10^{2} *to* 10^{4}	0.850	0.188
10^{4} *to* 10^{7}	0.480	0.250
10^{7} *to* 10^{12}	0.125	0.333

Morgan's equation is identical to McAdams' and incorporates a revision of the values of C and m based on more recent data.

Recently, Churchill and Chu [5] have published the following semi-empirical correlation equation for heat transfer by natural convection from horizontal cylinders:

$$Nu = 0.36 + 0.518 \left\{ \frac{Ra}{[1 + (0.559/Pr)^{9/16}]^{16/9}} \right\}^{0.25}. \tag{6}$$

Churchill and Chu state in their paper that equation (6) provides a good fit of representative data for all *Pr* and for $10^{-6} \leqslant Ra \leqslant 10^9$, except for the experimental data of Collis and Williams [6] for small diameter wires, which fall below equation (6) for $Ra < 10^{-6}$. They recommend that, for large temperature differences, the fluid properties in equation (6) be evaluated at the mean film temperature as a first approximation.

Raithby and Hollands [7] have recently published a correlation equation for laminar and turbulent natural convection for elliptic cylinders of arbitrary eccentricity for the case of constant surface temperature. Their equation assumes the following form for horizontal isothermal cylinders:

$$Nu^m = \left[\frac{2}{\ln\{1 + \pi 2^{3/4}/f_2^{3/4}\bar{C}_l Ra^{1/4}\}} \right]^m + (0.72C_t Ra^{1/3})^m \tag{7}$$

where

$m = 3.337$

$f_2 = 2.587$

$\bar{C}_l = (2/3)/[1 + (0.49/Pr)^{9/16}]^{4/9}$

$C_t = 0.14Pr^{0.084}$ or 0.15, whichever is smaller.

Raithby and Hollands do not state explicitly in their paper how the fluid properties are to be evaluated in equation (7)—it is presumed that evaluation at the mean film temperature is intended.

Traditionally, the fluid properties that appear in heat-transfer correlating equations have been evaluated at the mean film (reference) temperature t_f. Recently, however, several investigators have studied the question as to how the influence of temperature upon the physical properties of fluids should be taken into account for optimum correlation. Sparrow and Gregg [8], in a study of natural convection heat transfer from vertical plates, have recommended that the reference temperature be

$$t_r = t_s - r(t_s - t_b); \quad r = 0.38. \tag{8}$$

Fujii *et al.* [9], who conducted an experimental study of natural convection heat transfer from horizontal cylinders to water, spindtl oil and mobeltherm oil, have recommended that all fluid properties, except β, be evaluated at a reference temperature defined by

$$t_r = t_s - r(t_s + t_b); \quad r = 0.25 \tag{9}$$

and that β be calculated from

$$\beta = (\rho_b - \rho_s)/[\rho_f(t_f - t_b)]. \tag{10}$$

It is suggested here that all fluid properties be evaluated at a reference temperature defined by

$$t_j = t_b + j(t_s - t_b); \quad 0 \leqslant j \leqslant 1 \tag{11}$$

where j is selected so as to yield optimum correlation. The mean film reference temperature is a special case of t_j, because when $j = 0.5$, equation (11) reduces to equation (3).

A different method of evaluating fluid property variations as a function of temperature, which will be referred to here as the "correction factor" method, has been studied by several investigators for natural convection heat transfer. An example of this method is the work of Mikheyeva [10], who investigated the process of natural convection from horizontal tubes to air, water and oil. He has recommended the following correlation equation

$$Nu_b = 0.51 Ra_b^{0.25} (Pr_b/Pr_s)^{0.25}. \tag{12}$$

The term $(Pr_b/Pr_s)^{0.25}$ is a correction factor intended to account for the influence of property variation with temperature.

Fand and Keswani [11] have developed a method similar to Mikheyeva's to account for property variation with temperature, called the "surface correction exponent" method. The basic idea of this method, when applied to natural convection heat transfer, is to replace each dimensionless parameter that appears in a correlation equation, for example Pr, by Pr_p, where $Pr_p = Pr_s(Pr_b/Pr_s)^p$. (With this definition $Pr_{p=1} = Pr_b$ and $Pr_{p=0} = Pr_s$, and likewise for the other dimensionless parameters.) This method ensures that variation with temperature of all the dimensionless parameters that appear in a given correlation equation are accounted for. Thus, the correction exponent method constitutes a modification and generalization of Mikheyeva's correction factor.

The foregoing literature survey demonstrates that there exists a fundamental difference between the correlation proposed by Morgan and the correlations proposed by Kutateladze and others. Morgan's correlation states that Nu is only a function of Ra, whereas Kutateladze's correlation states that Nu is a function of Pr as well as Ra. The objective of the present study was to determine, on the basis of experimental data, which of these two approaches is the correct one; and, in the course of this study, to determine a new correlation equation or equations, if such are warranted, that optimumly account for property variation with temperature. The ranges of Ra and Pr selected for the present investigation cover approximately the ranges of Ra and Pr to which Kutateladze's equation (4) applies.

EXPERIMENTAL APPARATUS

The experiments with liquids reported herein were conducted inside a thermally insulated, stainless steel, circular tank having an ID of 7.659 in (19.454 cm) and a depth of 49.0 in (124.5 cm) into which various fluids (water and 20, 100 and 350 cS silicone oils) were introduced. An electrically heated, titanium, test cylinder having a diameter of 0.4555 in (1.157 cm) was used to furnish the heat-transfer data. The test cylinder was mounted horizontally in the tank between two vertical plates as shown in Fig. 1. The design of the test cylinder is described in detail in [11]. Briefly, it consists of three cylindrical sections, laid end to end, which are thermally insulated from each other. The central section, known as the "test section", is 1.000 in (2.540 cm) in length and it is this section that furnished the heat-transfer data. The end sections, called "guard sections", are 1.20 in (3.05 cm) long and their primary purpose is to prevent heat flow by conduction from the test section in the axial direction. During an experiment, each of the three sections was heated independently by supplying electric power, through a panel of auto-transformers, to the electrical heaters installed inside each section.

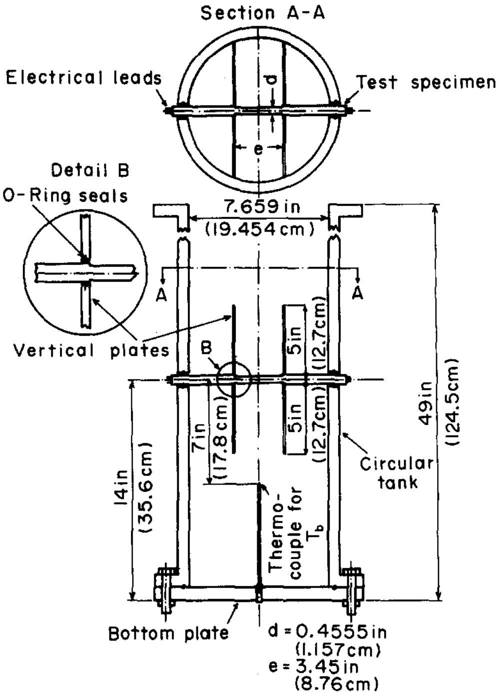

FIG. 1. Apparatus.

The test cylinder contains four embedded copper–constantan thermocouples. The test section contains two of these thermocouples which are orientated 90° apart relative to the axis of the cylinder, and each of the guard sections contains one thermocouple. Another thermocouple, positioned 7.0 in (17.8 cm) below the test cylinder, indicated the bulk temperature of the fluid. All the thermocouples were coupled through a selector switch to a potentiometer having a resolution of 0.05 degF (0.028 degC).

Since the thermocouples in the test cylinder are located below the surface of the cylinder, the temperature indicated by these thermocouples was greater than the actual surface temperature. Therefore, to determine the actual surface temperature, the radial temperature drop between the geometric center of the embedded thermocouples and the surface of the test cylinder was calculated by applying the one-dimensional conduction equation for heat transfer in the radial direction through a composite cylinder.

Power dissipations in the test section greater than or equal to 1 W (high power) were measured by a calibrated wattmeter. For power dissipations equal to or less than 1 W (low power), measurements of voltage and resistance were used to determine power dissipations in the test section. This was accomplished by experimentally determining in advance the electrical resistance of the test section heater coil as a function of temperature, so that power dissipation in the test section could be subsequently determined by simply measuring the voltage across the coil. The maximum error in the measurement of power dissipation was 1%.

Table 1. Experimental data for air from [13]; cylinder diameter = 0.7500 in (1.905 cm)

q[Btu/h · ft^2] (W/m^2)	t_s[degF] (degC)	t_b[degF] (degC)	Δt[degF] (degC)
34.08 (107.4)	89.72 (32.07)	60.32 (15.73)	29.40 (16.33)
68.17 (214.9)	112.53 (44.74)	61.40 (16.33)	51.13 (28.41)
102.6 (323.5)	128.64 (53.69)	57.44 (14.13)	71.20 (39.56)
136.7 (431.0)	147.79 (64.33)	57.70 (14.28)	90.09 (50.05)
170.8 (538.5)	170.96 (77.20)	63.08 (17.27)	107.88 (59.93)
204.8 (645.6)	187.27 (86.26)	60.59 (15.88)	126.68 (70.38)
272.7 (859.7)	220.43 (104.68)	61.62 (16.46)	158.81 (88.23)
306.7 (966.9)	234.42 (112.46)	61.09 (16.16)	173.33 (96.30)
340.8 (1074.4)	257.95 (125.53)	69.70 (20.94)	188.25 (104.58)
374.6 (1180.9)	265.40 (129.67)	60.80 (16.00)	204.60 (113.67)
408.6 (1288.1)	281.55 (138.64)	63.42 (17.46)	218.13 (121.18)
510.8 (1610.3)	330.90 (166.06)	74.33 (23.52)	256.57 (142.54)

The function of the two vertical plates in Fig. 1 was to constrain the flow in the neighborhood of the test cylinder to two dimensions. To substantiate the belief that the flow near the cylinder was two-dimensional, a comparison was made between the water data taken in the present apparatus and the water data taken by Fand and Keswani [12] with the same test cylinder but in a different apparatus. The previous investigators placed the test cylinder horizontally into a well insulated (wooden) rectangular tank containing water. The width of the tank, 3.5 in (8.89 cm), was equal to the entire heated length of the cylinder. The height and length of this tank were each greater than 100 cylinder diameters. Since the convective currents of the fluid were constrained from flowing in the axial direction by the sides of the tank, it was assumed that the natural convection flow in the tank approximated a two-dimensional flow past the test section of the cylinder. The experimental measurements of the temperature differences made by Fand and Keswani and those obtained in the present apparatus at approximately the same bulk temperatures are listed in the following table:

Q (W)	t_b (degC)	Δt (degC) [12]	Δt (degC) (present)	% difference
9.747	24.16	14.27	14.33	0.43
9.747	24.83	14.42	14.22	−1.37
19.628	23.72	24.11	24.44	1.37
19.628	23.44	24.07	24.50	1.77

The preceding tabulation shows that the average difference in Δt measured in the two different apparatuses under similar test conditions is 1%. Therefore, it was concluded that the present apparatus simulated a two-dimensional flow near the test cylinder.

The liquids used in the present study were mixed between tests, to obtain uniform bulk temperature, by means of circulating pumps and/or a motor-driven propeller-type stirrer.

The apparatus described above permitted the simultaneous measurements of t_s, t_b and Q. From this experimental data it was possible to calculate the Nusselt, Grashof and Prandtl numbers corresponding to the conditions of each test.

Twelve experimental air data points obtained by Fand [13] were included in the present study in order to extend the applicability of the final result to gases (air) in addition to liquids (water, oils). These air data are listed in Table 1.

EXPERIMENTAL PROCEDURE AND DATA

Since the local heat-transfer coefficient varies along the circumference of a horizontal cylinder, the temperature indicated by the two thermocouples in the test section of the cylinder used in this study with liquids were dependent on the angular position of these thermocouples relative to the convective plumes. Therefore, to determine the *average* heat-transfer coefficient, the following procedure was performed prior to the present experiments. With the tank filled with water, the test cylinder was rotated full-circle in 30° increments of arc and Δt* was measured at each station (12 in number) with the same power input Q to the test section and the bulk temperature of the fluid (water) held constant at 70 degF (21.11 degC). The mean value of Δt was determined utilizing Simpson's rule for numerical integration and it was found to occur at a particular angular position. The test cylinder was set into this angular position and left there for all subsequent measurements. This procedure assumes that the reading of Δt indicated by the thermocouples in this particular angular position is negligibly different from the mean value of Δt *for all* Q. This assumption is valid because it was found that the change in Δt along the circumference of the test cylinder was relatively small ($\Delta(\Delta t)/\Delta t \leqslant 0.027$), hence Δt is insensitive to small errors in the angular positioning of the thermocouples in the test section.

---

* In this context Δt stands for the average of the two surface temperatures indicated by the two thermocouples embedded in the test section of the heat-transfer cylinder minus t_b.

For water, the testing procedure began by letting the water "age" for 24 h under 5 atm pressure to dissolve all the air bubbles that had formed while the tank was being filled. This aging process was necessary because the presence of air bubbles on the test cylinder surface would have resulted in a spurious decrease in the heat-transfer coefficient. A frequent check was made to ensure that no bubbles were on the cylinder by viewing it through a window located in the tank wall.

After aging, the entire tank of water (still under pressure) was stirred thoroughly by means of a circulating pump for approximately 3 min. The purpose of this step was to attain a uniform bulk temperature. The water was then given approximately 10 min to come to rest after the pump was stopped. These preparations have been completed, a predetermined rate of power dissipation in the test section was created by appropriate adjustment of the electric power delivered to the test cylinder. The power to the guard sections was so adjusted that the temperatures of these sections were made as nearly equal to the average temperature of the two test section thermocouples as possible. Since in practice this equality could not be achieved absolutely, the temperature readings were recorded when the temperatures of the guard sections nearly "stradled" the average temperature of the test section, i.e. when one guard section temperature was higher and the other lower than the average temperature of the test section by equal amounts. This technique of taking data ensured that the heat loss or gain by conduction in the axial direction was negligible. Since the initial heating of the cylinder was a transient process, it was necessary to wait for some time before a steady state value of t_s was achieved. To ensure that the steady state was properly identified, temperature data were recorded throughout the transient process up to the desired steady state. Then the power was switched off and the water was circulated again in preparation for the next test. This procedure was repeated for different bulk temperatures and different rates of power dissipation in the test section.

For the three silicone oils, the same basic procedure as for water was followed for taking experimental data. Here, however, pressurizing the tank was not necessary since the likelihood of air-bubble formation was small, because air is almost insoluble in these oils. In addition to an oil circulating pump, a motorized, stainless steel stirrer was used to aid in the mixing of the viscous oils. The mixing and settling time allowed for the oils was approximately 25 min.

The approximate ranges of the various parameters for the five fluids studied here are listed below. In this table *Ra* and *Pr* have no subscripts, which implies that the temperature at which the properties are evaluated can range from the bulk fluid temperature to the surface temperature of the test cylinder. The overall ranges of *Ra* and *Pr* are: $2.5 \times 10^2 \leqslant Ra \leqslant 1.8 \times 10^7$ and $0.7 \leqslant Pr \leqslant 3090$. This range of *Ra* is approximately equal to that for which Kutateladze's equation (4) is applicable.

	20 cS oil	100 cS oil	350 cS oil	Water	Air
q (Btu/h-ft^2)	342—6780	342—6780	10—5060	700—34 500	34—510
(W/m^2)	(1080—21 300)	(1080—21 300)	(32—16 000)	(2200—109 000)	(110—1610)
t_s (degF)	86—233	92—270	73—270	80—200	90—330
(degC)	(30—112)	(33—132)	(23—132)	(23—93)	(32—160)
Δt (degF)	16—163	20—198	2—198	8—130	30—260
(degC)	(9—91)	(11—110)	(1—110)	(4—72)	(17—144)
$Ra \times 10^{-3}$	58—2260	12—610	0.25—176	0.11×10^3—18.0×10^3	9.8—98
Pr	55—200	190—940	625—3090	1.7—7.0	0.69—0.71

When the bulk temperatures of the liquids in the tank were higher than room temperature, there was a very slow flow of heat from the thermally insulated tank to the surrounding atmosphere. However slow, this flow of heat still caused a decrease in the bulk temperature. Thus, it is clear that for bulk temperatures greater than room temperature, there existed slight temperature gradients in the bulk of the fluid. These slight gradients caused weak spurious natural convection currents. Therefore, only those data for which t_b was within 2 degF (1.1 degC) of the room temperature were used for the final analysis. This temperature is called the "bulk-room temperature" hereinafter. The measurements made with bulk temperatures above room temperature were used to determine, with accuracy greater than could otherwise be as easily achieved, the data corresponding to the bulk-room temperature, as will be now explained. For each fluid, curves were drawn through the data points as shown, by way of example in Fig. 2. These curves were assumed to be the best representations of the experimental data, and the final data at bulk-room temperature were read from these lines. For the 20 cS oil, the bulk-room temperature was 70.0 degF (21.11 degC) and for the 100 cS, 350 cS oil and water, it was 72.0 degF (22.22 degC). The values of t and q corresponding to the bulk-room temperatures are recorded in Table 2. These data were considered to be the final experimental data.

Radiation heat transfer from the test section was calculated and found to be less than 0.1% of the power dissipated in any test and was, therefore, negligible. The maximum experimental error associated with any single determination of the heat-transfer coefficient h was estimated to be 4% for the air, water, 20 cS and 100 cS data, and 8% for the 350 cS data. The primary reason for the reduced accuracy in the case of the 350 cS oil is that the high viscosity of this oil makes it difficult to mix this oil sufficiently well so as to render its bulk temperature uniform.

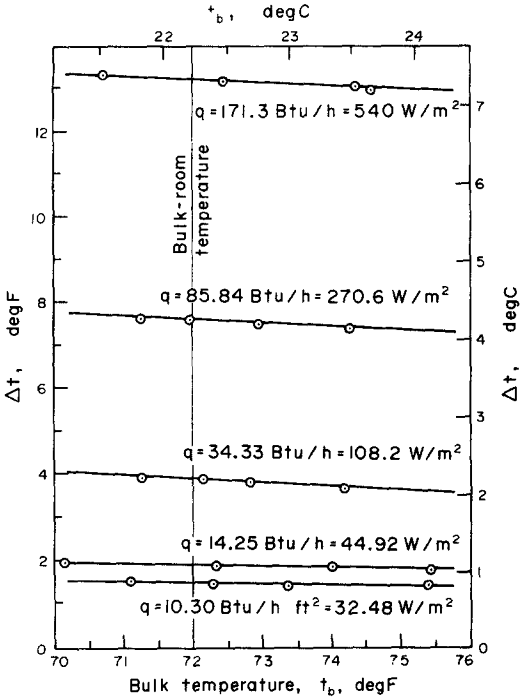

FIG. 2. Low power experimental data for 350 cS oil; cylinder diameter = 0.4555 in (1.157 cm).

Table 2. Experimental data for oils and water; cylinder diameter = 0.4555 in (1.157 cm)

q[Btu/h·ft²] (W/m²)	t_s[degF] (degC)	Δt[degF] (degC)
20 cS oil [t_b = 70.00 degF (21.11 degC)]		
339.9 (1071)	86.0 (30.00)	16.0 (8.89)
696.1 (2194)	98.0 (36.67)	28.0 (15.56)
1678.6 (5292)	126.3 (52.39)	56.3 (31.28)
3346.5 (10 550)	165.7 (74.28)	95.7 (53.17)
5057.0 (15 942)	201.2 (94.00)	131.2 (72.89)
6739.1 (21 245)	232.8 (111.56)	162.8 (90.45)
100 cS oil [t_b = 72.00 degF (22.22 degC)]		
339.9 (1071)	91.5 (33.06)	19.5 (10.84)
696.1 (2194)	106.2 (41.22)	34.2 (19.00)
1678.6 (5292)	141.2 (60.67)	69.2 (38.45)
3346.5 (10 550)	189.2 (87.33)	117.2 (65.11)
5057.0 (15 942)	232.2 (111.22)	160.2 (89.00)
6739.1 (21 245)	269.7 (132.06)	197.7 (109.84)
350 cS oil [t_b = 72.00 degF (22.22 degC)]		
10.30 (32.48)	73.5 (23.03)	1.5 (0.81)
14.25 (44.92)	73.9 (23.26)	1.9 (1.04)
34.33 (108.2)	75.8 (24.33)	3.8 (2.11)
85.84 (270.6)	79.6 (26.43)	7.6 (4.21)
171.3 (540.3)	85.3 (29.58)	13.3 (7.36)
339.9 (1071)	95.5 (35.28)	23.5 (13.06)
696.1 (2194)	113.0 (45.00)	41.0 (22.78)
1678.6 (5292)	156.2 (69.00)	84.2 (46.78)
3346.5 (10 550)	219.9 (104.39)	147.9 (82.17)
5057.0 (15 942)	270.2 (132.33)	198.2 (110.11)
Water [t_b = 72.00 degF (22.22 degC)]		
696.1 (2194)	80.0 (26.67)	8.0 (4.45)
1679 (5292)	87.6 (30.89)	15.6 (8.67)
3347 (10 550)	98.3 (36.83)	26.3 (14.61)
5057 (15 942)	108.3 (42.39)	36.3 (20.17)
6739 (21 245)	115.6 (46.44)	43.6 (24.22)
10144 (31 980)	132.2 (55.67)	60.2 (33.45)
13728 (43 277)	145.5 (63.06)	73.5 (40.84)
20622 (65 012)	169.5 (76.39)	97.5 (54.17)
27400 (86 380)	190.0 (87.78)	118.0 (65.56)
34512 (108 800)	211.8 (99.89)	139.8 (77.67)

DATA ANALYSIS AND RESULTS

The following three hypotheses for correlating natural convection heat transfer from horizontal cylinders were formulated in the course of the present study:

$$\text{I} \quad Nu_f = C Ra_f^{0.25} Pr_f^m \tag{13}$$

$$\text{II} \quad Nu_j = C Ra_j^{0.25} Pr_j^m \tag{14}$$

$$\text{III} \quad Nu_n = C (Gr_g Pr_p)^{0.25} Pr_p^m. \tag{15}$$

These hypotheses have the same algebraic form and differ only in the method used in each to evaluate fluid properties as a function of temperature. The validity of these hypotheses have been tested against the experimental data, as will be shown below. The previously published correlations of Kutateladze, Mikheyeva, Morgan, Churchill and Chu, and Raithby and Hollands have also been tested against the data. These published correlations have been compared with one another and with the three specific correlation equations that stem from the hypotheses indicated above.

In order to calculate the nondimensional parameters Nu, Gr, Pr and Ra that appear in the various correlation equations, it was necessary to have information concerning the physical properties ρ, ν, k, β and c_p of the several fluids investigated. The physical properties of air were obtained from [14, 15] and those of water from [16, 17]. The densities and viscosities of the three silicone oils studied here were measured experimentally, and their specific heats were obtained from [18] and from manufacturer's (Dow Corning) data. A formula for β was obtained from the experimentally measured values of ρ. A formula for the thermal conductivity of silicone oils was obtained from [19]. Appendix A contains all pertinent information concerning the physical properties of the three silicone oils employed in this study.

The parameters in each of the three hypotheses indicated above were determined so as to optimize the correlation of the experimental data. The optimum correlation equation corresponding to a given hypothesis was defined to be that specific equation which simultaneously yielded zero mean error and minimized the mean deviation relative to the entire body of data (air, water and three silicone oils).

To optimize Hypothesis I the exponent m was varied systematically in increments of 0.001 and, for each value of m, that value of the constant C which yielded zero mean error for all the data was calculated. Out of the many calculated results, one was selected as the optimum, namely, the one that minimized the mean deviation. The optimum specific correlation equation based upon Hypothesis I is as follows:

$$Nu_f = 0.474 Ra_f^{0.25} Pr_f^{0.047}. \tag{16}$$

This equation correlates the experimental data of all fluids combined with zero mean error and a mean deviation of 3.3% (E_{sd} = 4.2%). The errors incurred by equation (16) relative to each individual fluid and to all the data combined are given in Table 3.

Table 3. Statistical results

Equation	E_m	E_{md}	E_{sd}	Range of E
Air				
Kutateladze (equation 4)	13.0	13.0	2.1	29.5- 38.3
Mikheyeva (11)	11.7	11.7	1.5	9.6- 13.8
Churchill (6)	-12.9	12.9	2.0	-9.2--16.9
Raithby (7)	1.2	2.1	2.5	-4.1- 5.6
Morgan (5)	0.44	1.5	1.9	-4.4- 2.7
Hypothesis I (16)	-2.4	2.4	1.9	-0.8- -7.2
Hypothesis II (17)	-0.25	0.90	1.5	-4.2- 1.8
Hypothesis III (18)	-0.82	1.5	1.8	-3.6- 1.4
Water				
Kutateladze	11.8	11.8	3.2	29.8- 41.5
Mikheyeva	-0.4	1.3	1.4	-2.2- 1.6
Churchill	-2.8	2.8	1.3	-1.1- -3.8
Raithby	10.9	10.9	3.5	7.0- 17.7
Morgan	-0.64	2.3	2.9	-4.1- 4.5
Hypothesis I	5.2	5.2	1.9	2.5- 8.4
Hypothesis II	0.40	2.0	2.3	-2.7- 4.4
Hypothesis III	0.46	1.5	1.8	2.2- 3.0
20 cS oil				
Kutateladze	-9.6	9.6	2.2	5.7- 12.2
Mikheyeva	-6.1	7.7	7.7	-14.8- 4.2
Churchill	-12.5	12.5	1.1	-10.8--14.0
Raithby	-4.2	4.2	1.6	-6.4- -1.9
Morgan	-19.6	19.6	1.9	-17.1--22.0
Hypothesis I	0.08	0.91	1.2	-1.8- 1.4
Hypothesis II	0.21	0.39	0.54	-0.3- 1.1
Hypothesis III	0.55	0.63	0.59	-1.3- 0.2
100 cS oil				
Kutateladze	1.1	3.1	3.5	-3.1- 5.1
Mikheyeva	-11.1	11.3	9.2	-22.0- 0.4
Churchill	-16.7	16.7	1.7	-14.7--19.0
Raithby	-9.1	9.1	1.3	-11.2- -7.5
Morgan	-25.3	25.3	2.6	-22.4--28.4
Hypothesis I	-0.63	1.5	2.0	-3.3- 1.2
Hypothesis II	-0.38	0.60	0.71	-1.4- 0.6
Hypothesis III	-0.17	0.74	0.93	-1.2- 1.2
350 cS oil				
Kutateladze	-6.5	8.5	6.7	-12.6- 5.2
Mikheyeva	-21.6	21.6	11.7	-31.5- 0.5
Churchill	-18.6	18.6	4.0	-13.6--23.7
Raithby	-7.0	7.5	5.6	-13.3- 2.4
Morgan	-24.6	24.6	5.4	-16.2--29.6
Hypothesis I	-1.8	5.1	5.6	-8.3- 8.2
Hypothesis II	0.01	3.0	4.0	-5.4- 7.4
Hypothesis III	0.95	2.8	4.0	-4.5- 8.6
Combined				
Kutateladze	9.9	9.9	10.2	-12.6- 41.5
Mikheyeva	-4.2	11.0	14.1	-31.5- 13.8
Churchill	-12.2	12.1	6.2	-1.1--23.7
Raithby	-0.6	6.6	8.0	-13.3- 17.7
Morgan	-12.7	12.7	12.4	-28.4- 4.5
Hypothesis I	0.0	3.3	4.2	-8.3- 8.4
Hypothesis II	0.0	1.5	2.3	-5.4- 7.4
Hypothesis III	0.0	1.6	2.3	-4.5- 8.6

In a manner similar to that followed for Hypothesis I, the optimum specific correlation equation based on Hypothesis II was found to be:

$$Nu_j = 0.478 Ra_j^{0.25} Pr_j^{0.050} \tag{17}$$

where the fluid properties are evaluated at the reference temperature defined by:

$$t_j = t_b + j(t_s - t_b); \quad j = 0.32.$$

Equation (17) correlates the data of all fluids combined with zero mean error and a mean deviation of 1.5% ($E_{sd} = 2.3\%$). The errors incurred by equation (17) relative to each individual fluid and to all the data combined are given in Table 3.

The optimum specific correlation equation based upon Hypothesis III was found to be:

$$Nu_n = 0.456(Gr_g Pr_p)^{0.25} Pr_p^{0.057};$$

$$n = 0.20, \quad p = g = 0.50$$

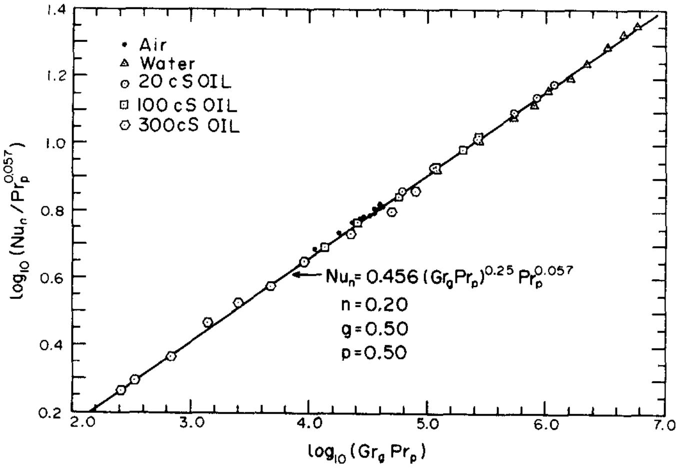

FIG. 3. Plot of experimental data and equation (18).

which may also be written as follows:

$$Nu_n = 0.456 Ra_y^{0.25} Pr_p^{0.057}; \quad n = 0.20, \quad p = y = 0.50. \tag{18}$$

Equation (18) correlates the data of all fluids combined with zero mean error and a mean deviation of 1.6% ($E_{sd} = 2.3\%$). The errors incurred by equation (18) relative to each individual fluid and to all the data combined are given in Table 3. A plot of equation (18) together with the experimental data is shown in Fig. 3.

DISCUSSION OF RESULTS

Table 3 contains statistics whereby the degree of agreement between the correlation equation of Kutateladze, Mikheyeva, Morgan, Churchill and Chu, Raithby and Hollands, and equations (16)–(18) with the experimental data considered here can be compared. The table shows that Morgan's correlation is in close agreement with the air data ($0.69 \leqslant Pr \leqslant 0.71$) and also with the water data ($1.7 \leqslant Pr \leqslant 7.0$), but it does not agree well with the oil data ($198 \leqslant Pr \leqslant 3090$). From this it is concluded that Morgan's correlation, which is dependent on Ra but not on Pr, does not adequately account for the influence of Pr on Nu. The same can be said of Mikheyeva's equation.

Churchill and Chu's correlation agrees fairly well with the water data, but exhibits a mean deviation of about 12% with respect to the entire body of data. For the calculations in Table 3, the fluid properties in Churchill's equation were also evaluated by the j-reference temperature method ($j = 0.32$) and by the correction exponent method ($n = 0.20$, $g = p = 0.50$), and the results were substantially the same as those recorded in Table 3.

Churchill and Chu's correlation, equation (6), differs from the others considered here inasmuch as it contains an additive constant, 0.36, which does not appear in any of the others. Such a constant, if it exists, can only be determined experimentally at Rayleigh numbers much lower than those studied here. Hence, the questions as to whether such an additive constant exists, and, if it does, what is its magnitude, cannot be answered on the basis of the data considered in this study.

Table 3 shows that the correlation proposed by Raithby and Hollands is in good agreement with all the data for all the fluids considered herein. This is remarkable in view of the fact that the Raithby–Hollands equation is applicable to the entire class of elliptic cylinders ranging from vertical flat plates (infinite eccentricity) to horizontal circular cylinders (zero eccentricity) over the wide range of Rayleigh numbers from 10^{-2} to 10^{12}. For the calculations in Table 3, the fluid properties in Raithby and Hollands' equation were evaluated at t_f. The properties in Raithby and Hollands' equation were also evaluated by the j-reference method ($j = 0.32$) and the correction exponent method ($n = 0.20$, $g = p = 0.50$), and the results were substantially the same as those recorded in Table 3.

Hypothesis I, per equation (16), exhibits a mean deviation of 3.3% relative to the entire body of data, which is good overall agreement. However, relative to the water data, this equation has a range of error from 2.5 to 8.4% (see Table 3) and a mean error and mean deviation of the error both equal to 5.2%, which implies that equation (16) overestimates Nu for the water data by 5.2% on the average. Although this level of agreement is usually considered adequate for practical calculations, the fact remains that these errors exceed the estimated maximum error of 4% in the experimental determination of h for the water data. Hence it is concluded that no correlation based upon Hypothesis I, which utilizes the mean film reference temperature, can represent the entire body of data within the limits of the experimental error.

As is indicated in Table 3, Hypotheses II and III, per equations (17) and (18), yield errors with respect to all the data that lie within the estimated experimental limits of error in the measurement of the heat-transfer coefficient. Therefore, it can be said that equations (17) and (18) represent the heat-transfer data to as high a degree of accuracy as may be reasonably expected of any correlation equation.

Kutateladze's correlation agrees quite well with the 100cS oil data, but it exhibits a mean deviation of about 10% with respect to the entire body of data.

However, the entries in Table 3 obscure a certain kind of agreement between Kutateladze's correlation and the entire body of data—this agreement is discussed in what follows. An interesting and revealing comparison can be made between Kutateladze's equation (4) and equation (16), which represents the data with adequate accuracy for practical applications ($E_m = 0$, $E_{md} = 3.3\%$, $E_{sd} = 4.2\%$—see Table 3). For this purpose equation (16) is written as follows:

$$Nu_f = F(Pr_f)Ra_f^{0.25} \tag{19}$$

where

$$F(Pr_f) = 0.474Pr_f^{0.047}.$$

The magnitude of $F(Pr_f)$ will now be compared with Kutateladze's recommended values for $K(Pr_f)$ in equation (4). If $F(Pr_f)$ is calculated at the end points of Kutateladze's low-*Pr* range, one obtains $F(Pr_f = 0.5) = 0.459$ and $F(Pr_f = 200) = 0.608$. The average of these two values of $F(Pr_f) = \frac{1}{2}(0.459+0.608) = 0.533$ and this number, 0.533, is very nearly identical to the value $K(Pr_f) = 0.54$ recommended by Kutateladze for the low-*Pr* range. Thus, if Kutateladze's value $K(Pr_f) = 0.54$ is regarded as an attempt to provide an appropriate *average* value of the coefficient of $Ra_f^{0.25}$, then there is close agreement between Kutateladze's equation and equation (16), and hence with the data. From this point of view equation (16) constitutes a refinement of Kutateladze's equation, inasmuch as $F(Pr_f)$ is a continuous function, as it must naturally be, whereas $K(Pr_f)$ is a 2-valued step function. Since Kutateladze did not provide an upper bound on the high-*Pr* range, probably due to a lack of data, it is not possible to make the same kind of comparison between his equation and equation (16) for the high-*Pr* range as was made above for the low-*Pr* range. However, if it is assumed that the upper bound of Kutateladze's high-*Pr* range is 3000, then the average value of $F(Pr_f)$ at the endpoints of the high-*Pr* range ($200 \leqslant Pr \leqslant 3000$) is 0.649, and this figure is nearly identical with Kutateladze's value $K(Pr_f) = 0.65$. Thus, one may conclude that a reasonable upper bound on Kutateladze's high-*Pr* range is 3000.

RESUME AND CONCLUSIONS

This empirical study has dealt with the rate of heat-transfer by natural convection from horizontal cylinders to air (gases), water and silicone oils (liquids) in the experimental ranges $2.5\times10^2 \leqslant Ra \leqslant 1.8\times10^7$ and $0.7 \leqslant Pr \leqslant 3090$. Three correlation hypotheses were postulated. Hypothesis I lead to the following correlation equation:

$$Nu_f = 0.474Ra_f^{0.25}Pr_f^{0.047} \tag{16}$$

where the fluid properties are evaluated at the mean film reference temperature. Equation (16) correlates the experimental data of all fluids combined with a mean deviation of 3.3% ($E_m = 0.0\%$, $E_{sd} = 4.2\%$).

The purpose of the second hypothesis was to find an optimum reference temperature for evaluating fluid properties as a function of temperature. Hypothesis II lead to the following correlation equation:

$$Nu_j = 0.478Ra_j^{0.25}Pr_j^{0.050} \tag{17}$$

where the fluid properties were evaluated at the reference temperature defined by:

$$t_j = t_b + j(t_s - t_b); \quad j = 0.32.$$

Equation (17) correlates the data of all fluids combined with a mean deviation of 1.5% ($E_m = 0.0\%$, $E_{sd} = 2.3\%$), which is an improvement over equation (15).

Hypothesis III encompassed the so-called "surface correction exponent" method of evaluating fluid properties as a function of temperature. This hypothesis lead to the following correlation equation:

$$Nu_n = 0.456Ra_y^{0.25}Pr_p^{0.057}; \quad n = 0.20; \quad p = y = 0.50. \tag{18}$$

Equation (18) correlates the data of all fluids combined with a mean deviation of 1.6% ($E_m = 0.0\%$, $E_{sd} = 2.3\%$). The errors incurred by equations (17) and (18) lie within the estimated experimental errors in the measurement of the heat-transfer coefficient.

The correlation equation recommended by Morgan, equation (5), states that *Nu* is a function of *Ra* but not *Pr*. However, equations (16)–(18) demonstrate that *Nu* is equal to $Ra^{0.25}$ multiplied by a function of *Pr*, in basic agreement with Kutateladze's correlation equation, equation (4).

The experiments studied here do not contain data at sufficiently low values of *Ra* to corroborate the existence of an additive correlation constant as exemplified by the number 0.36 in Churchill and Chu's equation. The first author of this paper has initiated an experimental investigation at low *Ra* in order to obtain additional information relative to the question of the additive constant.

The equation of Raithby and Hollands, with properties evaluated at t_f, yields a mean error of -0.6% ($E_{md} = 6.6\%$; $E_{sd} = 8.0\%$) relative to all the data considered herein. This relatively high level of agreement is remarkable in view of the fact that Raithby and Hollands' equation is applicable to the entire class of elliptic cylinders ranging from vertical flat plates (infinite eccentricity) to horizontal circular cylinders (zero eccentricity) over the wide range of Rayleigh numbers from 10^{-2} to 10^{12}.

The errors incurred by the three new correlation equations determined in this paper are less than are incurred by previously published correlations relative to the data studied here (see, particularly, the ranges of the error in Table 3). There is no priority given to equation (18), based on Hypothesis III, over equation (17), based on Hypothesis II, since both equations correlate the experimental data equally well.

The validity of equations (16)–(18) has been established for the ranges $0.7 \leqslant Pr \leqslant 3000$, $3\times10^2 \leqslant Ra \leqslant 2\times10^7$—these ranges cover many cases of practical interest. Since the use of equations (16)–(18) requires considerably less calculation effort than does the Raithby–Hollands' equation, especially in problems wherein the heat flux is specified and the resulting temperature difference is required, it follows that equations (16)–(18) can serve useful practical purposes.

It was pointed out above in the discussion of results that Morgan's correlation, equation (5), is in good agreement with the air data in the range $10^4 \leqslant Ra_f \leqslant 10^5$ analyzed in this study (see Table 3). Morgan's correlation is also in good agreement with the experimental data for air presented by Hesse and Sparrow [20] in the range $3 \times 10^2 \leqslant Ra_f \leqslant 10^4$. Now, as Ra decreases from 10^4 to 3×10^2 (the lower limit recommended for Ra_f in equation 16) the difference between the value of Nu_f predicted by Morgan's correlation and by equation (16) progressively increases from 3% at $Ra_f = 10^4$ to 19% at $Ra_f = 10^3$ and to 28% at $Ra_f = 3 \times 10^2$. Thus, both Morgan's correlation and equation (16) accurately represent different sets of data in the range $3 \times 10^2 \leqslant Ra \leqslant 10^4$ (Morgan's for air and equation (16) for oil), and yet these two correlation equations can differ significantly in their prediction for Nu_f *for the same value of* Ra_f in the range $3 \times 10^2 \leqslant Ra_f \leqslant 10^4$. This discrepancy cannot be attributed to the inclusion of the factor $Pr_f^{0.047}$ in equation (16), which does not appear in Morgan's equation, because for air this factor is very nearly equal to 1. An explanation for this discrepancy is offered in what follows.

In all the natural convection experiments with horizontal cylinders in air considered by Hesse and Sparrow in [20], the cylinder diameters were smaller by at least a factor of 10 than were the diameters of the cylinders that provided the experimental data analyzed in the present study. It is concluded here that the aforementioned discrepancy between Morgan's correlation and equation (16) stems from an experimental effect that depends upon cylinder diameter *but which cannot be accounted for by the Rayleigh number alone.* This conclusion has fundamental implications, for it means that the Nusselt number is a function of another dimensionless parameter, in addition to the Rayleigh and Prandtl numbers. The authors believe that this additional parameter is a function of *viscous dissipation*, which is usually neglected in the dimensional analysis of natural convection heat transfer. Gebhart [21] identifies such a viscous dissipation parameter in the form $g\beta dc_p^{-1}$.* Raithby and Hollands' equation does not solve this problem despite the fact that it represents the air data presented by Hesse and Sparrow more accurately than does equation (16) and despite the relatively low mean error that it yields with respect to the combined data considered herein (see Table 3). The Raithby and Hollands' equation comprises a judicious selection of algebraic forms and numerical constants which together constitute an excellent *compromise* between the trends of different sets of experimental data; thus for example, Table 3 shows that Raithby and Hollands' equation yields a *positive* error (that is greater than the experimental error) for every water data point, and it yields a *negative* error (that is greater than the experimental error) for every 100 cS oil data point—the net result is a relatively low mean error (−0.6%) for the combined data, but at the price of a range of error (−13.3–17.7%) that substantially exceeds the experimental errors. The practical import of the preceding discussion is that all presently available correlation equations, including equations (16)–(18), may be in error due to the omission of a viscous dissipation parameter; the magnitude of the possible error generally increases with decreasing Ra and may be as much as 20% at $Ra = 10^4$ and 30% at $Ra = 3 \times 10^2$. The first author has initiated a study to account for the effect of viscous dissipation in the correlation of natural convection heat transfer from horizontal cylinders.

Acknowledgements—The experimental measurements of the viscosities and densities of the silicone oils were performed by the Cannon Instrument Company, State College, PA 16801. The preliminary tests for locating the test cylinder in the proper angular position were performed by M. H. Lam, A. Sharma and J. A. Kerns performed some of the calculations. The authors wish to express their sincere thanks to Professor R. H. Sabersky for providing them with information concerning silicone oils.

REFERENCES

1. B. Gebhart, *Heat Transfer*, p. 37, 3rd edn. McGraw-Hill, New York (1971).
2. W. H. McAdams, *Heat Transmission*, 3rd edn. McGraw-Hill, New York (1954).
3. S. S. Kutateladze, *Fundamentals of Heat Transfer*, pp. 288–297. Academic Press, New York (1963).
4. V. T. Morgan, The overall convective heat transfer from smooth circular cylinders, *Adv. Heat Transfer* **11**, 199–210 (1975).
5. S. W. Churchill and H. S. Chu, Correlating equation for laminar and turbulent free convection from a horizontal cylinder, *Int. J. Heat Mass Transfer* **18**, 1049–1053 (1975).
6. D. C. Collis and M. J. Williams, Free convection of heat transfer from fine wires, Aust. Res. Lab. Aero., Note 140 (1954).
7. G. D. Raithby and K. G. T. Hollands, Laminar and turbulent free convection from elliptic cylinders with a vertical plate and horizontal circular cylinder as special cases, *J. Heat Transfer* **98**(1), 72–80 (1976).
8. E. M. Sparrow and J. L. Gregg, The variable fluid-property problem in free convection, *J. Heat Transfer* **80**, 879–886 (1958).
9. T. Fujii, M. Takeuchi, M. Fujii, K. Suzaki and H. Uehara, Experiments on natural convection heat transfer from the outer surface of a vertical cylinder to liquids, *Int. J. Heat Mass Transfer* **13**, 753–787 (1970).
10. M. Mikheyeva, *Fundamentals of Heat Transfer*, p. 79. Peace, Moscow (1968).
11. R. M. Fand and K. K. Keswani, The influence of property variation on forced convection heat transfer to liquids, *Int. J. Heat Mass Transfer* **15**, 1515–1536 (1972).
12. R. M. Fand and K. K. Keswani, Mass rate of flow in the natural convection plume above a heated horizontal cylinder immersed in a liquid, *J. Heat Transfer* **15**, 1175–1191 (1973).
13. R. M. Fand, The influence of sound on free convection from a horizontal cylinder in air, Ph.D. Thesis, Cornell University, 1959; see also R. M. Fand and J. Kaye, *J. Heat Transfer* **83**(2), 133–148 (1961).

---

*The value of the product βc_p^{-1} for air at elevated temperatures (900 degC) is approximately equal to the value of βc_p^{-1} for the oil experiments reported herein; therefore, the difference between the values of $g\beta dc_p^{-1}$ for these oil experiments and for the air experiments presented by Hesse and Sparrow are due primarily to differences in d.

14. Y. S. Touloukian, *Thermophysical Properties of Matter, Thermal Conductivity; Non-metallic Liquids and Gases*, Vol. 3, pp. 511–514. IFI/Plenum, New York (1970).
15. J. Hilsenrath, C. W. Beckett, W. S. Benedict, L. Fano, H. J. Hoge, J. F. Masi, R. L. Nuttall, Y. S. Touloukian and H. W. Wooley, *Tables of Thermodynamic and Transport Properties of Air*. Pergamon Press, Oxford (1960).
16. *1967 Steam Tables*. Edward Arnold, London.
17. J. C. S. Chou, Enthalphies of aqueous sodium chloride solution from 32 degF to 350 degF, *Desalination* **6**, 105–115 (1969).
18. R. C. Nielsen and R. H. Sabersky, Transient heat transfer in Benard convection, *Int. J. Heat Mass Transfer* **16**(12), 2407–2420 (1973).
19. O. K. Bates, Thermal conductivity of liquid silicones, *Ind. Engng Chem.* **41**, 1966–1968 (1949).
20. G. Hesse and E. M. Sparrow, Low Rayleigh number natural convection heat transfer from high-temperature horizontal wires to gases, *Int. J. Heat Mass Transfer* **17**, 796–798 (1974).
21. B. Gebhart, *Heat Transfer*, 3rd edn, p. 326. McGraw-Hill, New York (1971).

APPENDIX A

Thermophysical Properties of Silicone Oils

Kinematic viscosity (ν) *and density* (ρ)

Samples of the 20, 100 and 350 cS oil were subjected to experimental analysis with the following results:

Oil	t (degF), (degC)	ν (cS)	ρ (g/cm^3)
20 cS	77.0 (25.00)	19.62	0.9471
	130.0 (54.44)	11.99	0.9207
	180.0 (82.22)	8.125	0.8964
	230.0 (110.00)	5.781	0.8722
100 cS	77.0 (25.00)	103.0	0.9643
	130.0 (54.44)	61.30	0.9384
	180.0 (82.22)	40.72	0.9146
	230.0 (110.00)	28.50	0.8911
350 cS	77.0 (25.00)	349.6	0.9682
	130.0 (54.44)	207.4	0.9424
	180.0 (82.22)	137.3	0.9184
	230.0 (110.00)	95.94	0.8951

A regression analysis was performed on the preceding data to determine the coefficients in the following equations.

$$\ln(\ln z) = B_0 + B_1 \ln(t+460) + B_2[\ln(t+460)]^2 \qquad \text{(A.1)}$$

where

$$\nu = z - 0.7 \qquad \text{(A.2)}$$

and

$$\rho = A_0 + A_1 t + A_2 t^2 \qquad \text{(A.3)}$$

The constants in these equations are given in the following table:

	20 cS	100 cS	350 cS
A_0	0.985914	1.002980	1.0068264
A_1	−0.000509825	−0.000509895	−0.0005092
A_2	0.000000067	0.000000102	0.000000101
B_0	−14.411022	2.076379	7.082386
B_1	6.669645	1.053241	−0.70691767
B_2	−0.668427135	−0.181256535	−0.022038442

The values of t in the above equations are in degF while the dimensions of kinematic viscosity and density are cS and g/cm^3, respectively. The viscosity measurements are accurate to within 0.2% relative to the primary standard which is water at 68.0 degF (20.00 degC). The expected accuracy of the interpolated values of viscosity from equations (A.1) and (A.2) is 0.6%. Density measurements are accurate to within 0.1% and temperature measurements for the determination of the properties are accurate to within 0.01 degF.

Coefficient of thermal expansion (β)

By definition, the coefficient of thermal expansion is

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial t}\right)_p \qquad \text{(A.4)}$$

or,

$$\beta = \rho\left(\frac{\partial_1/\rho}{\partial t}\right)_p \qquad \text{(A.5)}$$

where v is the specific volume and $\rho = 1/v$. Hence

$$\beta = \frac{-(A_1 + 2A_2 t)}{(A_0 + A_1 t + A_2 t^2)}. \qquad \text{(A.6)}$$

Thermal conductivity (k)

The following equation, expressing k as a function of temperature, was obtained from [19]

$$k = 0.1661\frac{(4.66 + \nu_{25} - 0.000003\nu_{25}^2)}{(8.00 + \nu_{25})}(1 - 0.000601t) \qquad \text{(A.7)}$$

where t is in degC, ν_{25} is the kinematic viscosity at 25 degC and has dimensions of cS, and k has dimensions of W/m · degC. From the table of viscosity, the values corresponding to 20, 100 and 350 cS oil at 25 degC are 19.62 cS, 103.0 cS and 349.6 cS, respectively. Substitution of these values into equation (A.7) gives the following equations:

$$20\,\text{cS}: k = 0.1460(1 - 0.000601t) \qquad \text{(A.8)}$$

$$100\,\text{cS}: k = 0.1611(1 - 0.000601t) \qquad \text{(A.9)}$$

$$350\,\text{cS}: k = 0.1644(1 - 0.000601t) \qquad \text{(A.10)}$$

where t is in degC and k is in W/m · degC.

Specific heat (c_p)

The specific heat at constant pressure for the 20 and 350 cS oil as a function of temperature were determined from the following six data points supplied by the manufacturer.

	t(degF), (degC)	c_p(Btu/lb · degF)
20 cS	104.0 (40.00)	0.348
	212.0 (100.00)	0.353
	392.0 (200.00)	0.362
350 cS	104.0 (40.00)	0.335
	212.0 (100.00)	0.345
	392.0 (200.00)	0.353

These points plot approximately straight lines which can be expressed mathematically as:

$$c_p = c_{p_0} + \Delta c_p t. \qquad \text{(A.11)}$$

A least squares linear approximation was made on the manufacturer's data sets to determine the two unknown coefficients c_{p_0} and Δc_p for the 20 and 350 cS oils. The resulting equations are:

$$20\,\text{cS oil}: c_p = 0.34283 + 4.875 \times 10^{-5} t \qquad \text{(A.12)}$$

$$350\,\text{cS oil}: c_p = 0.33002 + 6.0660 \times 10^{-5} t \qquad \text{(A.13)}$$

where t is in degF and c_p has dimensions of Btu/lb · degF.

No specific data for c_p was available for the 100 cS oil.

Therefore, it was necessary to estimate this property. The average value of the Δc_p's in equations (A.12) and (A.13), namely, 5.4705×10^{-5} was taken to be the value of Δc_p for the 100 cS oil. The value of c_{p_0} for the 100 cS oil was determined with the aid of [18] which gave the value 0.335 Btu/lb · degF for the specific heat of 100 cS oil at 77.0 degF. This value of specific heat along with its corresponding temperature were substituted into equation (A.11) to determine the value of c_{p_0} for the 100 cS oil. The resulting equation for determining c_p for 100 cS oil is:

$$c_p = 0.330706 + 5.4705 \times 10^{-5} t. \qquad \text{(A.14)}$$

The conversion factor for c_p from British units to SI units is as follows: 1 Btu/lb · degF = 4.1868×10^3 J/kg · degC.

NOTE ADDED IN PROOF

A recent article by L. G. Napolitano, G. M. Carlomagno and P. Vigo, entitled "New classes of similar solutions for laminar free convection problems" [*Int. J. Heat Mass Transfer* **20**, 215–226 (1977)] demonstrates by theoretical means that, in general, laminar natural convection depends upon a dimensionless parameter (Eckert number) that includes the characteristic length of the heat-transfer surface, in addition to the Rayleigh and Prandtl numbers. This theoretical article supports the conclusion reached in the closing paragraph of the present paper, which conclusion was reached via the consideration of experimental data.

CONVECTION THERMIQUE NATURELLE AUTOUR DE CYLINDRES DANS L'AIR, L'EAU ET LES HUILES DE SILICONE, POUR DES NOMBRES DE RAYLEIGH ENTRE 3×10^2 ET 2×10^7

Résumé—On présente les résultats d'une étude expérimentale de la convection thermique naturelle autour de cylindres placés dans l'air, l'eau et trois huiles de silicone. Le domaine de variation du nombre de Rayleigh s'étend de $2{,}5 \times 10^2$ à $1{,}8 \times 10^7$ et le nombre de Prandtl varie de 0,7 à 3090. A partir des résultats expérimentaux donnés ici, on propose trois équations de corrélation. Ces équations ont la même forme algébrique et elles diffèrent seulement par la méthode utilisée dans chacune pour évaluer les propriétés du fluide en fonction de la température.

DER WÄRMEÜBERGANG BEI NATÜRLICHER KONVEKTION VON HORIZONTALEN ZYLINDERN AN LUFT, WASSER UND SILIKONÖLE FÜR RAYLEIGH-ZAHLEN ZWISCHEN $3 \cdot 10^2$ UND $2 \cdot 10^7$

Zusammenfassung—In dieser Arbeit wird über die Ergebnisse einer experimentellen Untersuchung des Wärmeübergangs bei natürlicher Konvektion von horizontalen Zylindern an Luft, Wasser und drei Silikonöle berichtet. Die Rayleigh-Zahlen wurden von $2{,}5 \cdot 10^2$ bis $1{,}8 \cdot 10^7$, die Prandtl-Zahlen von 0,7 bis 3090 variiert. Aufgrund der Meßwerte wurden drei Korrelationsgleichungen aufgestellt, welche denselben algebraischen Aufbau besitzen und sich lediglich in bezug auf die Wahl der Stoffwerte in Abhängigkeit von der Temperatur unterscheiden.

ПЕРЕНОС ТЕПЛА ОТ ГОРИЗОНТАЛЬНЫХ ЦИЛИНДРОВ К ВОЗДУХУ, ВОДЕ И СИЛИКОНОВЫМ МАСЛАМ ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В ДИАПАЗОНЕ ЧИСЕЛ РЭЛЕЯ 3×10^2 И 2×10^7

Аннотация — В данной статье приводятся результаты экспериментального исследования переноса тепла при естественной конвекции от горизонтальных цилиндров к воздуху, воде и трем сортам силиконовых масел. В ходе эксперимента число Рэлея изменялось от $2{,}5 \times 10^2$ до $1{,}8 \times 10^7$, а число Прандтля — от 0,7 до 3090. На основе полученных экспериментальных данных выведены три корреляционных уравнения. Эти уравнения имеют один и тот же алгебраический вид и отличаются только методом расчета свойств жидкости как функций температуры.